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Foundational Mathematical Errors that Affect Learning of Mathematics and Related Subjects in Higher Institutions

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Abstract: **Mathematics forms the foundation for logical reasoning, scientific inquiry, and technological advancement in higher education. However, many students entering higher institutions struggle with persistent foundational errors that significantly hinder their academic progress. This study examines the nature, prevalence, and impact of foundational mathematical errors that affect learning of mathematics and related subjects in higher institutions. Using a descriptive survey design, data were collected from 68 respondents through structured items covering number work, logical reasoning, contextual problem-solving, and creative sketching. The findings revealed strong performance in routine procedural tasks such as number notation (88% correct), but notable weaknesses in conceptual understanding, critical thinking, and application of knowledge to real-life contexts. For instance, only 50% correctly defined a composite number, while responses to contextual problems (e.g., boiling eggs and birds on a tree) showed over-reliance on assumptions and limited analytical reasoning. Additionally, 80% of respondents produced elementary-level sketches in a creativity task, indicating low development of spatial and design thinking skills. The findings suggest the need for pedagogical reforms that emphasize higher-order thinking, contextual learning, and learner-centered instructional strategies. The findings reveal that while many students demonstrate procedural competence in routine tasks, they exhibit deep-rooted misconceptions in fundamental areas such as number sense, operations, proportional reasoning, algebraic structure, and interpretation of mathematical language. Overall, the study highlights a dominance of algorithmic and examination-oriented learning approaches at the expense of deep conceptual understanding, interdisciplinary reasoning, and creativity.**

Keywords: **foundational errors, mathematical errors, conceptual skills, procedural competence.**

1. INTRODUCTION

1.1 Background of the study

Mathematics is generally considered as the most difficult subject to master by students at all levels from primary level to university level. Mathematics is the discipline which deals with numbers and their operations. It is full of abstract concepts, and it is not possible to achieve the aims of teaching and learning without understanding these concepts. Mathematics serves as a foundational discipline for scientific inquiry, technological advancement, logical reasoning, and analytical thinking in a higher education. Despite years of formal schooling, many students enter tertiary institutions with persistent foundational misconceptions in mathematics that significantly hinder their academic performance. This transition from secondary to tertiary education is frequently described as a period of academic disruption or “mathematical crisis” (Di Martino & Gregorio, 2019). Mathematics education plays a critical role in developing logical reasoning, analytical thinking, creativity, and problem-solving abilities.

Conceptual understanding in mathematics is the understanding of mathematical concepts and operations. Students demonstrate conceptual understanding in mathematics when they can recognize, label, and generate examples of concepts; use and inter relate models, diagrams, manipulative and varied representations of concepts; identify and apply principles;

know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. Beyond computational proficiency, effective mathematics instruction is expected to nurture conceptual understanding and the ability to apply knowledge to real-life situations. However, in many educational contexts, teaching and assessment practices often emphasize memorization, procedural fluency, and examination performance rather than deep understanding and critical thinking. Contemporary educational reforms advocate for learner-centered approaches that promote higher-order cognitive skills, interdisciplinary integration, and creativity. These challenges are not primarily due to lack of effort but rather to deeply rooted conceptual misunderstandings developed during earlier schooling. Research consistently shows that foundational errors are systematic misconceptions internally coherent but mathematically incorrect cognitive structures (Ay, 2017). When left unresolved, they accumulate and undermine performance in algebra, calculus, statistics, engineering, and other mathematically intensive disciplines.

One of the main goals of learning mathematics is to build the ability to understand concepts. Math material is hierarchical, so to learn a topic, students must first understand the material beforehand. This shows that learning mathematics is not enough to rely only on memorization, but requires deep understanding (Habinuddin & Binarto, 2020). Roswahyuliani (2022) emphasizes that the ability to understand mathematics is a basic skill that students must have, because it is an entrance to develop other cognitive abilities. In line with NCTM, mathematical understanding is included in one of the standards of mathematical thinking processes. Thus, this ability needs to be developed in a directed manner. In alignment with the National Council of Teachers of Mathematics (NCTM), mathematical understanding is recognized as one of the essential components within the standards of mathematical thinking processes. The NCTM emphasizes that true mathematical proficiency extends beyond procedural fluency to encompass a deep, conceptual understanding of ideas, relationships, and representations (NCTM, 2000). Such understanding enables learners to connect mathematical concepts, justify procedures, and apply knowledge flexibly across contexts.

Spatial reasoning, whether we realize it or not, is used in daily life. For example, when we use or read a map to find a location, compile pieces of the puzzle to be complete, draw a house design, and determine the position of each room. All of these activities use spatial reasoning. Spatial ability is even referred to as one of the keys to success in science, technology, engineering and mathematics, known as STEM. The growing emphasis on Science, Technology, Engineering, Arts, and Mathematics (STEAM) education further underscores the need for mathematical competence that goes beyond routine calculations. Learners are expected to interpret problems critically, question assumptions, reason logically, and apply knowledge in unfamiliar contexts. Despite these expectations, anecdotal evidence suggests that many students rely heavily on algorithmic thinking and fixed procedures when solving mathematical problems. For example, tasks that require interpretation of language, consideration of real-life variables, or creative representation frequently expose weaknesses in reasoning and conceptual depth. This often results in superficial understanding, especially when confronted with open-ended or contextual questions.

1.2 Problem Statement

Ultimately, understanding how learners respond to both routine and non-routine mathematical tasks provides valuable evidence for curriculum reform, teacher development, and assessment redesign aimed at producing mathematically competent and analytically minded graduates. Therefore, this ability must be developed in a structured and intentional manner through instructional practices that promote reasoning, problem-solving, and multiple representations of concepts. Despite possessing procedural mathematical knowledge, respondents demonstrate significant gaps in conceptual understanding, critical thinking, and application of mathematical concepts to real-life situations, highlighting a disconnect between mathematical knowledge and practical problem-solving skills. The present study was, therefore, designed to assess respondents' understanding across different dimensions of mathematical competence: number work, contextual reasoning, interpretation of mathematical expressions, problem-solving in everyday scenarios, and creative visualization.

1.3 Research Objectives

The following are the specific objectives that anchored the study:

(i) To assess respondents' conceptual understanding of basic number concepts such as composite numbers, place value, and number representation.

(ii) To examine respondents' ability to apply mathematical knowledge to real-life and contextual problem-solving situations.

(iii) To evaluate respondents' level of critical and analytical reasoning in interpreting mathematical expressions and word problems.

(iv) To assess respondents' creativity and spatial visualization skills through drawing and design-based tasks.

1.4 Research Questions

The following research questions were used to find answers to the specific objectives:

(i) What is the level of respondents' conceptual understanding of basic number concepts such as composite numbers and place value?

(ii) How effectively do respondents apply mathematical knowledge to contextual and real-life problems?

(iii) To what extent do respondents demonstrate critical and analytical reasoning when interpreting mathematical expressions and word problems?

(iv) What is the level of respondents' creativity and spatial reasoning as reflected in open-ended drawing tasks?

1.5 Significance of the study

The findings of this study have significant implications for mathematics education. They suggest that the current approach to mathematics education, which emphasizes rote memorization and examination-driven instruction, is inadequate. Instead, a learner-centered, concept-based approach that fosters analytical thinking, flexibility, and real-world problem-solving skills is needed. To address these gaps, educators should prioritize conceptual understanding, encourage critical thinking, and provide opportunities for students to apply mathematical concepts to real-life situations. By doing so, we can equip students with the skills necessary to tackle complex problems and navigate an increasingly complex world. Ultimately, this study highlights the need for a paradigm shift in mathematics education, one that prioritizes deep conceptual understanding, creativity, and problem-solving skills. By working together, educators, policymakers, and stakeholders can create a more effective and engaging mathematics education system that prepares students for success in the 21st century.

2. LITERATURE REVIEW

2.1 Conceptual understanding of basic number concepts.

Conceptual understanding of basic number concepts involves an integrated, functional grasp of mathematical ideas rather than isolated procedural skills, allowing students to recognize, represent, and apply principles like cardinality and part-part-whole relationships. Research highlights that this foundational understanding is critical for future math performance and involves moving from concrete, experiential learning to abstract, flexible number usage. In the mathematics education literature, the term conceptual understanding was more established than in other contexts. Kilpatrick, Swafford, and Findell (2001) defined conceptual understanding as "an integrated and functional grasp of mathematical ideas, students with conceptual understanding know more than isolated facts and methods" (p. 118).

Concepts are the building blocks of knowledge (Charlesworth, 2012). Conceptual understanding and procedural knowledge are essential to the development of problem-solving skills (Geary, 2004). These skills contribute towards the effective processing of information when solving problems. The five strands of mathematical proficiency (Kilpatrick, Swafford, and Findell, 2001) represent the interdependence of the five components of a learner's proficiency in mathematical problem-solving. The use of problem-solving to teach mathematics does not only develop knowledge and skills but also helps students make sense of it. They will be able to see how new concepts connect to their existing knowledge which help them to be mathematically proficient (Mayer, 2008). Conceptual knowledge is in general abstract knowledge that addresses the essence of mathematical principles and relationships between them, while procedural knowledge consists of symbols, conditions, and processes that can be applied to complete a given mathematical task (Hiebert and Lefevre, 1986). Procedural knowledge is meaningful only if it is connected to a theoretical fact. Faulkenberry (2003) suggests that conceptual knowledge is rich with relationships and refers to the basic mathematical constructs and relationships between the ideas that illustrate mathematical procedures, and give them meaning. On the other hand, procedural knowledge addresses the mastery of mathematical skills, and acquaintance with the procedures to determine the mathematical components, algorithms, and definitions. Many researchers suggest that both conceptual knowledge and procedural knowledge are important components in understanding mathematics (Desimone et al., 2005; Hiebert et al., 2005).

2.2 Applications of mathematical knowledge to contextual and real-life problems.

Mathematics learning is a learning process that aims to improve mathematical thinking skills and the ability to construct new knowledge so that students can analyze various problems and improve their logical and systematic thinking skills (Salim & Pitriani, 2021; Syahna et al., 2022). The National Council of Teachers of Mathematics (NCTM) explains that students must master five key skills, namely mathematical problem solving, mathematical communication, reasoning, connecting, and mathematical representation (Mubarika et al., 2020). Problem solving in mathematics learning plays an important role (Gholami, 2023; Hidayah et al., 2020). The National Center for Education Statistics (NCES) states that problem solving is one of the skills that must be mastered because it helps a person select and execute the right solution in a problem solving strategy (Al-Mutawah et al., 2019). In its implementation, the problem-solving process involves reasoning and proof, communication, connection, and mathematical representation. The problem-solving process is always associated with the way students think and reason, and places greater emphasis on analytical, critical, logical, systematic, and creative thinking skills (Ekayana et al., 2020; Rahayu, 2019).

The objective of mathematics lessons in school is to equip students with the skills to understand mathematical materials in solving mathematical problems and to relate factual material to everyday life (BSKAP, 2025). There are three cognitive activities involved in the problem-solving process, namely 1) problem presentation, which consists of recalling prior knowledge and identifying the objectives and initial conditions relevant to the given problem; 2) problem-solving identification, which consists of setting goals and developing action plans to achieve those goals; and 3) solution implementation, which consists of executing action plans and evaluating the results (Anam et al., 2018). Providing problems related to everyday life is one way to improve problem-solving skills (Astuti and Amin, 2019). Problems related to everyday life are called contextual problems (Lestari et al., 2021). All forms of problems and mathematical problems related to the real world or abstract concepts that are context-specific and cannot be solved using conventional methods are referred to as contextual problems (Amin et al., 2021; Ekayana et al., 2020; Rizki, 2018). Students are required to understand and implement concepts in solving everyday problems, which are manifested through contextual mathematical problems or those commonly presented in the form of story problems (Fadilah and Bernard, 2021). Thus, contextual problems can be defined as all forms of real problems in everyday life that students have experienced, are experiencing, or will experience in accordance with the context.

2.3 Critical and analytical reasoning behind mathematical expressions and word problems

The literature on critical and analytical reasoning in mathematics emphasizes a shift from procedural fluency (rote calculation) to conceptual understanding, where learners must interpret, evaluate, and justify mathematical expressions and word problems. Research indicates that critical thinking math word problems serve as vital tools for developing higher-order thinking, enabling students to decipher ambiguous data, recognize patterns, and apply mathematical concepts to real-world scenarios. Critical thinking is an intellectual process within the sphere of a person's cognitive dimension in active reasoning. In essence, critical thinking is a process of reasoning (Elder & Paul, 2008). Based on a definition widely adopted nowadays, critical thinking is a reasonable and reflective thinking, that is focused on deciding what to believe or do (Ennis, 2018). Thus, considering the importance of reasoning, the foremost expected outcome in all types of mathematics learning is thinking and reasoning skills (Animasaun & Abegunrin, 2017). It was explicitly stated in a framework of the National Council of Teachers of Mathematics (NCTM) that reasoning is the foundation of mathematics teaching, because it is not enough for students just to understand and remember facts, and the development of critical thinking skills is absolutely necessary for them to succeed in mathematics learning (National Council of Teachers of Mathematics, 2000).

According to NCTM, mathematical interpretation involves logical conclusions based on evidence, and this is similar to the concept of critical thinking according to the perspective of other experts (Elder & Paul, 2008; Ennis, 2018). The focus on reasoning becomes important in the context of mathematics teaching in the classroom. In practice, it usually depends on the choice of tasks and the valuable learning experience in developing reasoning, including a classroom environment that supports it so that teachers can set up the learning discourse effectively and perform assessments accordingly to monitor the progress of students' reasoning (National Council of Teachers of Mathematics, 2000). Maulyda (2020) stated that each stage of the learning process should be assessed with the aim to measure the success rate of the learning process carried out, as well as the targeted goals. It was also stated in her study that the assessment at each stage should meet certain criteria, as well as the indicators specified as part of the reflection of the learning success. Finally, a student's progress in reasoning or critical thinking can be measured through an assessment of his or her critical level of thinking.

2.4 Creativity and spatial reasoning in open-ended mathematical drawing tasks

Creativity and spatial reasoning are deeply intertwined in open-ended mathematical drawing tasks, particularly in geometry, where they enable students to move beyond routine, algorithmic procedures to produce novel, varied solutions. These tasks, which allow for multiple solutions or strategies, foster divergent thinking (the ability to generate diverse, creative, and original ideas) while relying on spatial visualization and mental rotation to manipulate shapes. Spatial reasoning is commonly used in mathematics, especially in geometry but not only in geometry but also in other fields of mathematics. For example, in arithmetic, to understand the concept of 2 + 3, children will count it by imagining using concrete objects like they have two apples and then given three more apples so that the number of apples they have now is five. This mental process occurs in their minds. The National Council of Teachers of Mathematics even states that mathematical reasoning is one of the five mathematical abilities students must possess, especially spatial reasoning which is very closely related to geometry (NCTM, 2000). Children use spatial skills in understanding the world, for example visualizing the relationships between objects around them and arranging it through spatial activities. Embedding spatial skills in the mathematics curriculum can have a positive impact on the learning process and mathematics education (Forndran, Lowrie and Harris, 2019). Several studies show that students with good spatial reasoning also have better results in mathematical assessment and their spatial reasoning will help them to understand the concepts of the material being taught (Winarti, 2018).

There are three important components in spatial reasoning namely, spatial visualization, mental rotation and spatial orientation (Lowrie and Logan, 2018). Spatial visualization is a mental ability to imagine and manipulate images using different perspectives (Mix, Levine, Cheng, Young, and Hambrick, 2016). This activity allows people to predict what will happen when they do visual activities such as folding and rearranging the pieces (Yenilmez, 2015). Mental rotation is a cognitive process in which a person imagines how 2-dimensional and 3-dimensional objects will appear after they have turned a point with a certain angle (Ramful, Lowrie and Logan, 2016). Spatial orientation is almost the same as mental rotation, objects are manipulated mentally but with a fixed observer's point of view. In other words, imagine how an object looks from a different perspective from the observer. In addition to spatial reasoning, creativity is also important (Cho, 2017; Suh and Cho, 2020) and has even become a major concern in psychology, one of which focuses on the mental process of creating creative ideas (Blazhenkova and Kozhevnikov, 2016).

In mathematics, creativity is also needed to process information that we already have to solve problems. Guildford in his three-dimensional structure model states that aspects of creativity consist of fluency, flexibility, originality, and elaboration (He, 2017). Fluency (the number of meaningful ideas) is related to the number of ideas that can be given in a short time. Flexibility (number of different response categories), namely the ability to think flexibly in seeing a variety of different perspectives. Originality (originality of the responses given) is the ability to think in unusual ways to generate new ideas and approaches used in problem-solving (Vale, Pimentel and Barbosa, 2018; Kenett et al., 2018). Elaboration is the ability to imagine and explain something in detail (He, 2017). According to Novitasari et al. (2021) spatial reasoning is applicable in the form of spatial visualization and mental rotation in creativity to solve problems. It is worth noting that spatial reasoning has been used in daily life, such as in building designs, determining routes and using maps. Spatial reasoning is also used in mathematics, especially in geometry.

3. RESEARCH METHODOLOGY

3.1 Research Design

The study employed a quantitative research approach, specifically utilizing a descriptive survey design. The quantitative paradigm was considered appropriate because the study sought to measure and analyse respondents' performance objectively through numerical data. This approach enabled the researcher to determine the prevalence and patterns of foundational mathematical errors among first-year university entrants. The design allowed for systematic data collection, statistical analysis, and objective interpretation of results without researcher interference in the testing process.

3.2 Population and Sample

The target population for the study comprised all first-year entrants admitted into the university for the academic year under review. First-year students were selected because they represent fresh entrants from various pre-university educational backgrounds, making them suitable for assessing foundational mathematical competencies acquired prior to university admission. A sample of sixty-eight (68) first-year students was selected for the study. The sampling technique employed was simple random sampling to ensure that each student had an equal chance of being selected. This approach minimized selection bias and enhanced the representativeness of the sample.

3.3 Research Instrument

Data were collected using a Mathematics Diagnostic Test (MDT) designed by the researcher. The test consisted of randomly structured questions covering key foundational mathematical concepts, including: Number operations; Basic geometry; Word problems and Logical reasoning. The items were constructed to assess respondents' conceptual understanding, procedural knowledge, spatial, and problem-solving ability rather than mere memorization. The instrument comprised short structured-response questions to allow for comprehensive assessment of students' reasoning processes.

3.4 Validity and Reliability of the Instrument

To ensure content validity, the test items were carefully aligned with the core mathematics competencies expected at the pre-university level. Subject experts in mathematics education reviewed the instrument to verify clarity, relevance, and appropriateness of the questions. A pilot test was conducted with a small group of students who were not part of the main study sample. Feedback from the pilot study led to minor modifications in question wording and structure to enhance clarity. Reliability of the instrument was established using internal consistency measures. The test items demonstrated acceptable reliability for educational assessment purposes.

3.5 Data Collection Procedure

The diagnostic test was administered under standardized examination conditions within the university premises. Prior to the administration of the test, participants were briefed on the purpose of the study and assured that the results would be used strictly for academic research purposes. Students were given adequate time to carefully read, analyze, and respond to all questions. The duration of the test was structured to ensure that time pressure did not significantly influence performance outcomes. Invigilation was conducted to maintain academic integrity and ensure independent work. Completed scripts were collected immediately after the allocated time had elapsed.

3.6 Data Analysis

Descriptive statistics such as Frequencies and Percentages were used to analyze respondents' performance patterns and identify areas of difficulty. The results were presented using tables to facilitate clear interpretation of findings. Error patterns were analyzed to determine common misconceptions and foundational gaps in mathematical understanding.

3.7 Ethical Considerations

Ethical principles were strictly observed throughout the study. Participation was voluntary, and students were informed that their involvement would not affect their academic standing. Anonymity and confidentiality of respondents were ensured by assigning identification codes instead of using names.

4. DATA PRESENTATION AND ANALYSIS

4.1 Preliminary Observations

The analysis of the data reveals several important patterns regarding respondents' conceptual understanding, reasoning ability, and interpretation of mathematical and contextual problems.

4.2 Numeration skills

The study sought to gauge the level of appreciation of the respondents regarding their numeration skills. They were required to define a 'composite number' and write a number in words and another in figures. The response is shown in the Table 1 below:

Table 1: Number work

s/n	Item	No. Correct	No. Incorrect
1	What is a composite number? (N=68)	34 (50%)	34 (50%)
2	Give an example of a composite number (N=68)	46 (68%)	22 (32%)
3	Write the following number in words: 480,032 (N=68)	60 (88%)	8 (12%)
4	Write in figures: Forty-two thousand and eighteen (N=68)	60 (88%)	8 (12%)

The data shows that only 50% of the respondents could correctly define a composite number, although 68% could give an example of what a composite number is. The respondents' performance in number notation was, however, very strong.

About 88% correctly wrote numbers in words and figures, indicating high competence in basic place value and numeration skills. However, weak understanding of composite numbers suggests procedural familiarity without deep conceptual understanding.

4.3 Critical contextual thinking

(a) If an egg is boiled to cook in 5 minutes, how long will it take to boil 10 of such eggs?

This question was an emotional one because it was the basis for which most school children were caned without understanding the chemistry behind their punishment. Teachers usually use this question for mental drills in basic schools without even explaining the reasoning behind the "correct" response, which, in most cases, was "the same 5 minutes". By critical analysis, in appreciating this question, one needs to know if the same bowl, same amount of water, and same source of heat (temperature), that would be used to boil the 10 eggs.

Table 2: The boiling Time

S/n	Item description	No. of Respondents	Percentage
1.	5	51	76%
2.	5<x<50	2	3%
3.	50	14	21%
	Total	67	100%

From the Table 2 above, 76% of the respondents stated that it would take 5 minutes to boil 10 eggs, assuming simultaneous cooking without considering variables such as water quantity and heat capacity. While 76% of the respondents believed that it will take the same 5 minutes to boil all 10 eggs, they are blamed of not taking note of the size of the container, the water quantity and the enthalpy change of the ten eggs. In practical sense, the quantity of water needed for boiling an egg will not be the same for the 10 eggs. This means that if the heat source remains the same, then more time is required for more water quantity to boil. The other 21% of the respondents, responded from the other extreme end. They said 50 minutes. Perhaps, they assumed that each egg would be boiled separately. This thought process is not economical hence unwelcome for a higher level student in university. Apparently, only 2 out of the 68 representing 3% of the respondents had the idea that the time would be more than 5 minutes but definitely less than 50 minutes. This response is the view of the researcher. This shows limited application of scientific reasoning and poor integration of mathematical logic with real-life physical concepts.

(b) Division versus Sharing

Many students usually take sharing to mean division in mathematics. To them two people sharing 4 items is treated as 4 divided by 2 which gives 2. The study again attempts to find out how the respondents appreciated the difference between division and sharing. This suggests rigid algorithmic thinking rather than contextual reasoning. The study also identified evidence of rigid algorithmic thinking among respondents.

Table 3: Sharing illustrations

s/n	Sharing formula	Response	Percentages
1	4,0	4	6%
2	3,1	12	18%
3	2,2	48	72%
4	1,3	2	3%
5	0,4	2	3%
		68	**100%**

From Table 3, approximately 72% interpreted the concept of "sharing" strictly as mathematical division. While division is indeed related to sharing in many contexts, the automatic association of sharing with division indicates a lack of flexibility in mathematical thinking. In many real-life situations, sharing may involve estimation, proportional reasoning, or other forms of distribution that do not necessarily correspond directly to a simple division algorithm. The tendency to default immediately to a memorized procedure suggests that students rely heavily on algorithms without evaluating whether they are appropriate for the context of the problem. The remaining 28% provided alternative distributions (4,0; 3,1; 1,3; 0,4), reflecting broader interpretation of the context. Rigid reliance on algorithms is a common consequence of traditional mathematics instruction that emphasizes procedural accuracy and speed. When students are trained primarily to apply formulas and rules, they may become less inclined to explore alternative interpretations or reasoning strategies. As a result, their mathematical thinking becomes mechanical rather than analytical.

4.4 Difference between 2x5 and 5x2 (A diagrammatic illustration, if any)

In elementary mathematics education, multiplication is often introduced as repeated addition or grouping. Understanding this conceptual representation is important because it supports later learning in areas such as algebra, arrays, and mathematical modeling.

Table 4: Response showing difference between 2x5 and 5x2

s/n	Question	No. of respondents	Percentages
1	There is no difference	55	81%
2	There is difference	13	19%
		68	**100%**

The results in Table 4 reveal an overemphasis on answers rather than on understanding mathematical structures and relationships. A significant majority of respondents (81%) indicated that the expressions 2 times 5 and 5 times 2 were equivalent (equal). While it is mathematically correct that both expressions yield the same product due to the commutative property of multiplication, the responses suggest that many students focused only on the final answer rather than recognizing the conceptual distinction between “two groups of five” and “five groups of two.” The reality is that the respondents were all unable to demonstrate diagrammatically, the conceptual difference between these two concepts. The respondents’ inability to recognize the structural difference between these representations suggests that their understanding of multiplication may be limited to procedural calculation rather than conceptual interpretation. Such weaknesses may hinder students’ ability to grasp more advanced mathematical ideas that depend on structural reasoning.

4.5 The hunter and birds on a tree

It is worth noting that this question did not state the way the bird was killed; whether gun fire (loud noise) or catapult (silent shot). Again, the type of birds on the tree was also not stated. This is because not all types of birds will fly away just by hearing gunfire. Again, if catapult or normal stone was thrown at the bird, others might not hear or notice it (Table 5).

Table 5: Response on reaction of birds on a tree after killing one

s/n	No. of birds remaining	Response	Percentages
1	None (0)	61	90%
2	Nine (9)	7	10%
		68	**100%**

About 90% of the respondents stated that there would be no (0) birds left on the tree. They explained that after killing the one by the hunter, the rest would fly away. This assumption by the respondents revealed how many students usually answer examinations questions based on their mindset instead of reading the question thoroughly. Meanwhile, 7 out of the 68 representing 10% of the respondents were of the view that 9 birds would still remain on the tree since they might not be aware of the killing process. These responses imply that the students based their responses on assumptions not stated in the question, indicating weak critical reading and over-reliance on personal experiences rather than textual evidence. Inasmuch as mathematics is an experiential subject, critical consciousness is required through careful reading of its questions.

4.6 Spatial understanding of the respondents

The response to the above question was a mix of several stages of the respondents’ levels of education; from KGs to the university levels.

Table 6: Sketching of furniture

s/n	Item	Response	Percentages
1	KG2	7	10%
2	P1	54	80%
3	HIGHER	7	10%
		68	**100%**

A whooping 54 out of the 68 respondents representing 80% made sketches that were similar to those usually drawn by Primary one pupils. Typical of their sketches are the usual four stands (legs) of rectangular tables. Only 10% made drawings that exhibited their maturity. These people made sketches that had round top tables, one stand tables, two-stand tables, and tables of higher order. The remaining 10% made drawings that were not legible. That is, their drawings could not be interpreted as tables, but could only be likened to those drawn by KG children. The findings (Table 6) point to the urgent need for a shift from rote, examination-driven instruction toward a learner-centered, concept-based, and creativity-enhancing mathematics education approach that fosters analytical thinking, flexibility, and real-world problem-solving skills. This lack of flexibility in thinking was further evident in respondents' simplistic and immature sketches of furniture, indicating a lack of creativity and analytical thinking.

4.7 Discussion of results

The study assessed respondents' numeration skills, critical contextual thinking, and spatial understanding through a series of questions. The results showed that while respondents performed well in number notation, they struggled with conceptual understanding of composite numbers. Only 50% could correctly define a composite number, and 68% could provide an example, indicating a gap between procedural familiarity and deep conceptual understanding. The study also revealed weaknesses in critical contextual thinking. When asked about boiling eggs, 76% of respondents assumed simultaneous cooking without considering variables like water quantity and heat capacity. This lack of consideration for real-world factors suggests a limited application of scientific reasoning and poor integration of mathematical logic with physical concepts.

Furthermore, the study found that respondents often focused on answers rather than understanding the underlying mathematical structure. For instance, 81% of respondents believed that 2x5 and 5x2 were equivalent, focusing on the product rather than the conceptual difference between "2 groups of 5" and "5 groups of 2". This reflects an overemphasis on rote memorization rather than conceptual understanding. The study also highlighted rigid algorithmic thinking, with 72% of respondents interpreting "sharing" as mathematical division. This lack of flexibility in thinking was further evident in respondents' simplistic and immature sketches of furniture, indicating a lack of creativity and analytical thinking.

The implications of these findings suggest that higher institutions cannot assume mastery of basic concepts among incoming students. Instead, there is a need for diagnostic assessment, bridging programmes, and instructional strategies that emphasize conceptual understanding, reasoning, and problem-solving rather than rote procedures. The findings point to the urgent need for a shift from rote, examination-driven instruction toward a learner-centered, concept-based, and creativity-enhancing mathematics education approach that fosters analytical thinking, flexibility, and real-world problem-solving skills.

5. SUMMARY OF FINDINGS, CONCLUSIONS & RECOMMENDATIONS

5.1 Summary of Findings

The findings highlight the need for a shift from rote, examination-driven instruction to a learner-centered, concept-based approach that fosters analytical thinking, flexibility, and real-world problem-solving skills.

Specifically, the following are some of the findings of the study:

1. Respondents performed well in routine number representation tasks but struggled with conceptual, analytical, and interpretative questions.
2. Many respondents failed to consider underlying assumptions, contextual variables, and alternative interpretations in problem-solving situations.
3. A significant number interpreted questions mechanically (e.g., sharing as division, multiplication as product only) rather than exploring conceptual meaning.
4. Real-world application problems (eggs, birds) exposed weaknesses in connecting mathematics with scientific and logical reasoning.
5. The furniture sketch task revealed limited creative thinking and poor development of spatial imagination among most respondents.

5.2 Conclusions

In conclusion, study reveals that while students may display some procedural competence in mathematics, significant gaps remain in conceptual understanding, contextual reasoning, and creative thinking. Addressing these weaknesses requires deliberate educational reforms, including diagnostic assessment, bridging programmes, and innovative teaching strategies that emphasize conceptual learning and problem-solving. Such reforms are essential for preparing students to meet the cognitive demands of tertiary-level mathematics and related disciplines. Another important implication is the need to strengthen critical reading and interpretation skills in mathematics. Many mathematical errors arise not from computational weaknesses but from misinterpretation of problems. Students must therefore be trained to read questions carefully, identify key information, and analyze the structure of problems before attempting to solve them.

Ultimately, the study underscores the importance of developing a mathematics education approach that promotes analytical thinking, creativity, and conceptual understanding. Mathematics should not be viewed merely as a collection of formulas and procedures but as a system of reasoning that helps individuals interpret and solve real-world problems. By adopting instructional practices that encourage exploration, reasoning, and contextual application, educators can help students develop the intellectual skills necessary for success in higher education and professional fields.

5.3 Recommendations

To address the gaps in mathematical understanding and problem-solving skills, the study proposes several key recommendations. Firstly, educational authorities should strengthen conceptual teaching approaches, emphasizing meaning and understanding of mathematical expressions before procedures. This can be achieved through increased use of manipulatives, visual models, and real-life examples to deepen understanding of concepts such as multiplication and composite numbers.

Secondly, critical thinking must be integrated into mathematics instruction. This can be done through the use of open-ended questions that require justification and explanation, encouraging students to question assumptions and consider multiple possibilities while solving mathematics questions.

Thirdly, mathematics teachers should promote interdisciplinary learning by integrating basic scientific principles into mathematics teaching and encouraging cross-curricular problem-solving approaches. For instance, concepts like heat transfer can be incorporated into mathematical problems to enhance understanding.

Additionally, learners should be trained to approach mathematics with a reading comprehension mindset, analyzing the exact wording of questions, identifying implicit assumptions, and recognizing missing information. This skill is crucial for effective problem-solving and can be developed through targeted instruction.

Furthermore, mathematics learning should incorporate creative and design thinking, reflecting the discipline's connection to engineering and design. Teachers can include tasks such as drawing, modeling, and design-based activities to foster creativity and spatial visualization skills.

Finally, educational administrators should organize workshops on conceptual teaching strategies, and teachers should allocate marks for reasoning, explanation, and process involved in solving mathematics questions, moving beyond answer-focused marking schemes. By implementing these recommendations, mathematics education can become more effective, fostering deeper understanding, critical thinking, and problem-solving skills.

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